



# **DYNAMIC ANALYSIS OF SPACE TETHER MISSIONS**

**Volume 126  
ADVANCES IN THE ASTRONAUTICAL SCIENCES**

**by  
Eugene M. Levin**

*Published for the American Astronautical Society by  
Univelt, Incorporated, P.O. Box 28130, San Diego, California 92198  
Web Site: <http://www.univelt.com>*

Copyright 2007

by

AMERICAN ASTRONAUTICAL SOCIETY

AAS Publications Office  
P.O. Box 28130  
San Diego, California 92198

Affiliated with the American Association for the Advancement of Science  
Member of the International Astronautical Federation

*First Printing 2007*

Library of Congress Card No. 57-43769

ISSN 0065-3438

ISBN 978-0-87703-537-4 (Hard Cover)

ISBN 978-0-87703-538-1 (Soft Cover)

Published for the American Astronautical Society  
by Univelt, Incorporated, P.O. Box 28130, San Diego, California 92198  
Web Site: <http://www.univelt.com>

Printed and Bound in the U.S.A.

# CONTENTS

Foreword . . . . .	iii
About the Book . . . . .	9
<b>NEW MISSIONS FOR SPACE TETHERS . . . . .</b>	<b>11</b>
<b>1. HARPOONING A COMET</b>	
<i>COMET NUCLEUS SAMPLE RETURN</i> . . . . .	19
Nomenclature . . . . .	20
1.1. Race to Comets and Asteroids . . . . .	21
1.2. Tether-Mediated Sample Retrieval . . . . .	24
1.3. Spacecraft Motion in the Vicinity of the Comet . . . . .	26
1.3.1. <i>Dynamic Models and Equations</i> . . . . .	26
1.3.2. <i>Hovering and Close Approach Trajectories</i> . . . . .	29
1.4. Equations of Tethered Motion . . . . .	34
1.4.1. <i>Equations in a Newtonian Form</i> . . . . .	34
1.4.2. <i>Equations in Minakov's Form</i> . . . . .	36
1.4.3. <i>Equations for the Tangent Orientation Angles</i> . . . . .	39
1.4.4. <i>Calculation of the Tether Tension</i> . . . . .	42
1.4.5. <i>Normalization of the Arclength</i> . . . . .	44
1.5. Tether Deployment . . . . .	45
1.5.1. <i>Tether Tension Range</i> . . . . .	45
1.5.2. <i>Deployment Dynamics</i> . . . . .	47
1.5.3. <i>Energy Considerations</i> . . . . .	49
1.6. Tether Retrieval . . . . .	52
1.6.1. <i>Retrieval Dynamics</i> . . . . .	52
1.6.2. <i>Dynamics of a Capsule Moving Along the Tether</i> . . . . .	55
1.6.3. <i>Other Scenarios</i> . . . . .	58
1.6.4. <i>Tether Safety</i> . . . . .	59
1.7. Conclusions . . . . .	60

**2. SURVIVING IN SPACE**

<i>TETHER PHYSICS AND SURVIVABILITY EXPERIMENT</i>	63
Nomenclature	64
2.1. The Story of Survival	65
2.2. Evolution Model	69
2.2.1. <i>Equations of Motion</i>	69
2.2.2. <i>Internal Friction in the Tether</i>	71
2.2.3. <i>Pendular Motions</i>	72
2.2.4. <i>Forms of Small Oscillations</i>	74
2.2.5. <i>Modal Decomposition</i>	76
2.2.6. <i>Equations of Libration</i>	77
2.2.7. <i>Evolution Equations</i>	78
2.3. Non-Resonant Motions	80
2.4. Resonant Motions	83
2.4.1. <i>Poincare Maps and Resonance Zones</i>	83
2.4.2. <i>Evolution in the Resonance Zone</i>	87
2.5. Deployment Dynamics	91
2.5.1. <i>Deployment and the Resonances</i>	91
2.5.2. <i>Tether Bow During and After Deployment</i>	94
2.6. Thermal Effects	97
2.6.1. <i>Thermal Hysteresis</i>	97
2.6.2. <i>Thermal Cycles and Libration Evolution</i>	99
2.7. Rotation of the End-Bodies	101
2.7.1. <i>Alignment of the Spin Axis</i>	101
2.7.3. <i>Torsional Oscillations</i>	103
2.7.2. <i>Interaction with Transverse Waves in the Tether</i>	104
2.8. Conclusions	105

**3. SAVING A SPACE STATION**

<i>MIR ELECTRODYNAMIC TETHER SYSTEM</i>	107
Nomenclature	108
3.1. Space Station in Trouble	109
3.2. Electrodynamical System Model	112
3.2.1. <i>Equations of Motion</i>	112
3.2.2. <i>Tether Heating</i>	114
3.2.3. <i>Electric Current in the Tether</i>	115
3.2.4. <i>Geomagnetic Field Model</i>	116
3.2.5. <i>Calculating Geomagnetic Field Along the Orbit</i>	118
3.3. Pendular Motions	120
3.3.1. <i>Equations of Pendular Motion</i>	120
3.3.2. <i>Operation Modes</i>	123
3.3.3. <i>System Characteristics</i>	124

3.4. Librational Instability . . . . .	126
3.4.1. <i>Out-of-plane Instability</i> . . . . .	126
3.4.2. <i>In-plane Instability</i> . . . . .	131
3.5. Libration Control . . . . .	132
3.6. Attitude Dynamics of the Space Station . . . . .	134
3.6.1. <i>Equations of Mir's Rotation</i> . . . . .	134
3.6.2. <i>Relative Equilibrium</i> . . . . .	139
3.6.3. <i>Small Oscillations</i> . . . . .	140
3.7. Resonant and Non-Resonant Motions . . . . .	142
3.7.1. <i>Limitations on Tether Libration AMplitudes</i> . . . . .	142
3.7.2. <i>Spectral Analysis of Free Oscillations</i> . . . . .	143
3.7.3. <i>Spectral Analysis of Forced Oscillations</i> . . . . .	145
3.7.4. <i>Non-Resonant Motions</i> . . . . .	147
3.7.5. <i>Resonant Motions</i> . . . . .	148
3.8. Attitude Control . . . . .	151
3.8.1. <i>Gyro-Damping</i> . . . . .	151
3.8.2. <i>Attitude Control with Thrusters</i> . . . . .	154
3.9. Transverse Oscillations of the Tether . . . . .	154
3.9.1. <i>Equations of Transverse Oscillations</i> . . . . .	154
3.9.2. <i>Linear Oscillations</i> . . . . .	157
3.9.3. <i>Modal Decomposition</i> . . . . .	159
3.9.4. <i>The Effect of Transverse Oscillations on Mir's Rotation</i>	164
3.9.5. <i>Controlling Transverse Oscillations</i> . . . . .	164
3.10. Torsional Oscillations of the Tether . . . . .	166
3.10.1. <i>Rotation of the End-Body</i> . . . . .	166
3.10.2. <i>Equations of Torsional Motion</i> . . . . .	168
3.10.3. <i>Torsional Eignefrequencies</i> . . . . .	169
3.10.4. <i>Evolution of Motion</i> . . . . .	170
3.10.5. <i>Thermal Effects</i> . . . . .	171
3.11. Longitudinal Damper . . . . .	173
3.12. Tether State Estimation . . . . .	174
3.13. Conclusions . . . . .	179
<b>4. GOING FOR A SPIN</b>	
<i>ELECTRODYNAMIC DELIVERY EXPRESS</i> . . . . .	183
Nomenclature . . . . .	184
4.1. Perpetuum Tether Mobile . . . . .	185
4.2. Dynamic Model . . . . .	187
4.2.1. <i>Newtonian Formulation</i> . . . . .	187
4.2.2. <i>Motion of the Center of Mass</i> . . . . .	189
4.2.3. <i>Tether Rotation</i> . . . . .	190
4.2.4. <i>Small Oscillations</i> . . . . .	191
4.2.5. <i>Minakov's Formulation</i> . . . . .	193

4.3. Operation of a Spinning Tether System . . . . .	196
4.3.1. <i>Electrodynamic Thrust</i> . . . . .	196
4.3.2. <i>Magnetic Field Along the Orbit</i> . . . . .	198
4.3.3. <i>Evolution of the Spin Axis</i> . . . . .	199
4.3.4. <i>Tether Oscillations and Stability</i> . . . . .	202
4.3.5. <i>Control of the Spinning Tether System</i> . . . . .	205
4.4. Orbit Transfers with Spinning Tethers . . . . .	208
4.4.1. <i>Equations of Orbit Evolution</i> . . . . .	208
4.4.2. <i>Optimal Current Scheduling</i> . . . . .	209
4.4.3. <i>Average Performance Characteristics</i> . . . . .	215
4.4.4. <i>Fastest Orbital Plane Change</i> . . . . .	218
4.4.5. <i>Orbital Plane Change Assisted by Altitude Change</i> . . . . .	224
4.4.6. <i>Fastest In-Plane Orbit Transfers</i> . . . . .	226
4.4.7. <i>Elliptic Orbits with High Apogees</i> . . . . .	230
4.5. Attitude Motion of the End-Bodies . . . . .	230
4.6. Conclusions . . . . .	233

## 5. CATCHING A PAYLOAD

<i>MOMENTUM EXCHANGE ELECTRODYNAMIC REBOOST SYSTEM</i> . . . . .	235
Nomenclature . . . . .	236
5.1. Electrodynamic Tether Transport . . . . .	237
5.2. Dynamic Model . . . . .	241
5.2.1. <i>Equations of Motion</i> . . . . .	241
5.2.2. <i>Gravitational Field Model</i> . . . . .	242
5.3. Stationary Rotation . . . . .	245
5.4. Natural Forms of Oscillations . . . . .	247
5.4.1. <i>Equations of Small Oscillations</i> . . . . .	247
5.4.2. <i>The Eigenvalue Problem</i> . . . . .	248
5.4.3. <i>Eigenforms and Eigenvalues</i> . . . . .	249
5.5. Decomposition of Motion . . . . .	252
5.5.1. <i>General Equations</i> . . . . .	252
5.5.2. <i>Calculation of the Integrals of Tension</i> . . . . .	254
5.5.3. <i>The Case of a Perfectly Tapered Tether</i> . . . . .	255
5.5.4. <i>Coupled Longitudinal-Transverse Oscillations</i> . . . . .	257
5.6. Gravitational Forces . . . . .	258
5.6.1. <i>Expansion of the Newtonian Term</i> . . . . .	258
5.6.2. <i>General Power Series Expansion</i> . . . . .	260
5.6.3. <i>Contribution of the Higher Order Terms</i> . . . . .	261
5.7. Precision Simulation Model . . . . .	262
5.8. Minakov's Formulation . . . . .	263
5.8.1. <i>Equations of Motion in Minakov's Form</i> . . . . .	264
5.8.2. <i>Boundary Conditions</i> . . . . .	266

5.8.3. <i>Motion of the Center of Mass</i>	267
5.8.4. <i>Quasi-Static Tension</i>	268
5.8.5. <i>Simulation Approach</i>	271
5.9. Resonant Excitation of Oscillations	273
5.10. Non-Gravitational Perturbations	274
5.10.1. <i>Method of Computation</i>	274
5.10.2. <i>Aerodynamic Forces</i>	275
5.10.3. <i>Electrodynamic Forces</i>	275
5.10.4. <i>Solar Radiation Pressure</i>	275
5.10.5. <i>Thermal Expansion of the Tether</i>	276
5.10.6. <i>Creep in the Tether</i>	277
5.10.7. <i>Tether Mass Loss</i>	277
5.11. Estimation and Control Approaches	280
5.12. Conclusions	280

## 6. LISTENING TO THE ECHOES OF THE PAST

<i>SUBMILLIMETER PROBE OF THE EVOLUTION OF COSMIC STRUCTURE</i>	283
Nomenclature	284
6.1. Observatories in Space	285
6.2. Tethered Synthetic Aperture Interferometer	288
6.3. Diamond Shape Configurations	290
6.3.1. <i>Symmetry and Dynamic Balance</i>	290
6.3.2. <i>Synthetic Aperture Coverage</i>	292
6.4. Dynamic Model	295
6.4.1. <i>Equations of Motion</i>	295
6.4.2. <i>Gravitational and Non-Gravitational Forces</i>	297
6.5. Stationary Motions	299
6.5.1. <i>Quasi-Rigid Motions</i>	299
6.5.2. <i>Dynamic Symmetry and Integrals of Motion</i>	300
6.5.3. <i>Tether Tension in Stationary Motion</i>	303
6.6. Periodic Motions	305
6.7. Controlled Motions	307
6.7.1. <i>Tether Tension Control</i>	307
6.7.2. <i>Controlling the Diamond Shape Configuration</i>	309
6.7.3. <i>Changing the Spin Plane</i>	310
6.7.4. <i>Residual Out-of-Plane Oscillations</i>	312
6.7.5. <i>Variable Diamond Shape with Fixed Tether Lengths</i>	313
6.8. Tether Oscillations	314
6.8.1. <i>Tether Design and Thermal Regime</i>	314
6.8.2. <i>Longitudinal Oscillations</i>	315
6.8.3. <i>Transverse Oscillations</i>	315
6.8.4. <i>Torsional Oscillations</i>	317

6.8.5. <i>The Effect of Tether Oscillations on the End-Bodies</i>	318
6.9. Simulation Model	319
6.9.1. <i>Fast and Slow Components of Motion</i>	319
6.9.2. <i>Difficulties of Numeric Integration</i>	320
6.9.3. <i>Relative Motion of the Tether</i>	322
6.9.4. <i>Calculation of Tether Tension</i>	326
6.9.5. <i>Modal Expansion</i>	329
6.9.6. <i>Simulation Approaches</i>	330
6.10. Conclusions	331
<b>7. COLONIZING THE MOON</b>	
<i>LUNAR TRANSPORTATION SYSTEMS</i>	333
Nomenclature	334
7.1. The Earth-Moon Highway	335
7.2. Lunar Space Elevator Revisited	337
7.2.1. <i>Transfer Trajectories to the Lunar Elevator</i>	337
7.2.2. <i>Equations of Motion and Relative Equilibrium</i>	340
7.2.3. <i>Characteristics of Tether Configurations</i>	341
7.2.4. <i>Non-Equatorial Configurations</i>	345
7.2.5. <i>Fail-Safe Elevator</i>	348
7.3. Rotating Docking Port on the Moon	349
7.3.1. <i>The Concept</i>	349
7.3.2. <i>Dynamic Features of the Rotating Port</i>	352
7.4. Conclusions	356
<b>8. FLYING IN FORMATION</b>	
<i>TRIANGULAR FORMATIONS OF TETHERED SATELLITES</i>	359
Nomenclature	360
8.1. Constellations, Clusters, and Formations	361
8.2. Triangular Tethered Formations	365
8.2.1. <i>Dynamic Model</i>	365
8.2.2. <i>Quasi-Rigid Rotation</i>	368
8.2.3. <i>Tether Tension</i>	369
8.2.4. <i>Symmetrical Formations</i>	370
8.3. Rotation and Precession	372
8.3.1. <i>Spin Stabilization</i>	372
8.3.2. <i>Precession of the Spin Axis</i>	376
8.3.3. <i>Conical Precession</i>	378
8.3.4. <i>Tidal Evolution</i>	383
8.4. Electrodynamic Control	386
8.4.1. <i>The effect of Ampere Forces</i>	386
8.4.2. <i>Optimal Thrust</i>	389
8.4.3. <i>Electromagnetic Torque</i>	392



8.5. Tether Oscillations . . . . .	395
8.6. Tetrahedron Formations . . . . .	399
8.7. Conclusions . . . . .	400
<b>9. SETTING SAIL</b>	
<i>ELECTRODYNAMIC SAIL CONCEPT</i> . . . . .	403
Nomenclature . . . . .	404
9.1. Large Space Structures . . . . .	405
9.2. Electrodynamic Sail . . . . .	408
9.3. Sail Dynamics and Control . . . . .	411
9.3.1. <i>Orbital Motion and Quasi-Rigid Rotation</i> . . . . .	411
9.3.2. <i>Electrodynamic Thrust</i> . . . . .	413
9.3.3. <i>Electrodynamic Torque</i> . . . . .	414
9.3.4. <i>Modelling Sail Deformations</i> . . . . .	416
9.3.5. <i>Performance Characteristics</i> . . . . .	418
9.3.6. <i>Rendezvous with a Spinning Sail</i> . . . . .	420
9.4. Conclusions . . . . .	421
References . . . . .	425
Subject Index . . . . .	449
List of Tables . . . . .	453